# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2019**

**16/17/18UST3AL01 – MATHEMATICAL STATISTICS - I**

Date: 06-11-2019 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

### Section A

**Answer all questions** **10x2=20**

1. Define a random experiment.
2. Two unbiased dice are thrown. Find the probability that both the dice show the same number.
3. Define distribution function of a random variable
4. Let X be a random variable with following probability distribution

| X | -3 | 6 | 9 |
|---|---|---|---|
| P(X=x) | 1/6 | 1/2 | 1/3 |

   Find E(X) and E($X^2$).

5. Determine the parameters of binomial distribution whose mean is 4 and variance is 3.
6. Derive the moment generating function of rectangular distribution.
7. Define the smallest order statistic.
8. Derive the mean of Poisson distribution.
9. Write the probability density function of F-distribution.
10. List any three properties of characteristic function.

### Section B

**Answer any five questions** **5x8=40**

11. State and prove addition theorem of probability.
12. From a city population, the probability of selecting (i) a male or a smoker is 7/10 (ii) a male smoker is 2/5 (iii) a male, if the smoker is already selected is 2/3. Find the probability of selecting (a) a non- smoker (b) a male (c) a smoker if a male is first selected.
13. The joint probability density function of a two-dimensional random variable (X,Y) is given by f(x,y)=2; 0<x<1, 0<y<x. (i) Find the marginal density functions of X and Y (ii) Find the conditional density functions and (iii) Check for independence of X and Y.
14. Show that the exponential distribution lacks memory.
15. Two independent random variables X and Y are both normally distributed with means 1 and 2 and standard deviations 3 and 4 respectively. If Z=X-Y, write the probability density function of Z. Also state the median, standard deviation and mean of the distribution of Z. Find P(Z+1≤0)

16. In a continuous distribution whose relative frequency density is given by $f(x)= y_0 . x(2-x)$, $0 \leq x \leq 2$. Find mean and variance.
17. Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y):

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

18. Derive the joint probability density function of a group of k order statistics.

## Section C

**Answer any two questions** **2x20=40**

19. (a) State and prove Bayes' theorem.

    (b) A and B throw alternatively with a pair of balanced dice. A wins if he throws a sum of six points before B throws a sum of seven points, while B wins if he throws a sum of seven points before A throws a sum of six points. If A begins the game, show that his probability of winning is 30/61.

    (10+10)
20. (a) Define beta variate of second kind. Obtain its mean and variance.

    (b) The daily consumption of milk in a city, in excess of 20,000 litres, is approximately distributed as a gamma variate with parameters a=1/10,000 and $\lambda$=2. The city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day? (12+8)
21. (a) State and prove Chebychev's inequality.

    (b) Derive the probability density function of Chi-Square distribution. (10+10)
22. (a) State and prove Central Limit Theorem

    (b) Define a t-distribution. Derive the moments of t-distribution. (10+10)

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